Different States, Choice, Structure and Aggregation in Simulated Social Networks

Rob Stocker

School of Information Technology and Electrical Engineering
&
Defence and Security Applications Research Centre
The Australian Defence Force Academy

Abstract

The fabric of society lies in the networks of connections and patterns of communication that its members create deliberately or inadvertently. Information, ideas, values and norms are passed across this fabric and members can form aggregates or allegiances that centre on common interests, goals, attitudes and the like. Simple multi-agent models of social networks have provided useful insights into the emergence of global network behavior where agents have limited or binary choices of state. This paper examines the impact that a greater number of choices of state have on the emergence of clusters of agents. It examines the global behavior of static populations of interacting computational agents, connected in fixed networks structures that are faced with multiple choices of state. Results indicate that aggregation around a few states appears to be a universal property, independent of network structure.

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Please address correspondence to Bob Stocker at r.stocker@adfa.edu.au.

Introduction

How does the process of socialization occur? How are society's values, norms, and ideals cemented in social structure? How do rumours, urban legends and myths spread in society? Why are some people better informed than others? How do people form into groups? What influences the outcome of political elections? These questions create strong interest for social network researchers. Typical of complex adaptive systems, social networks exhibit clustering behavior that may be based around a common principle.

Complex systems are characterised by large numbers of components that interact and communicate through patterns of connections called networks (Holland 1995). Complex systems behavior is at the root of many natural and artificial phenomena. However, knowledge and understanding about their behavior, design and management remains largely empirical. The complexity of multi-agent systems means that traditional methods of studying them are not effective (Bura *et al.* 1995; Gilbert & Troitzsch 1999; Goldspink 2002; Wassermann 1980), as they tend to be unstable and unpredictable (Bak & Sneppen 1993; Erdös & Rènyi 1960; Green 1993; Horgan 1995; Langton 1990).

Structural analysis (Berkowitz 1982; Freeman 1989; Hammer 1979; Hummon & Carly 1993; Wellman 1988) investigates sets of relationships that exist in the complex interactions of social members in the context of the social system in which they act (Erickson 1988; Lorrain & White 1971; Scott 2000). Analogous to emergent properties and phase changes common in complex systems, such social transitions result from network topology and information exchange between connected network members (Bura *et al.* 1995; Fliedner 2001; Holyst *et al.* 2000; Klüver & Schmidt 1999; Schecter 2002). Network models provide a natural and effective means of representing hierarchical levels in social systems (Fliedner 2001; Hammer 1979; Hummon & Carly 1993; Sanil *et al.* 1995; Wassermann & Faust 1995). Graph theory (a mature research discipline) can be used to draw maps or topologies of social structures including random graphs, scale-free *al.* 2000; Albert & Barabási 2001; Barrat & Weigt 2000; Doreian 1979; Erdös & Rènyi 1960; Jeong *et al.* 2000; Wassermann 1980; Watts & Strogatz 1998). The dynamic behavior of multi-agent network systems is typically modeled using general rules that describe the behavior of the agents, and topological rules that describe the patterns by which agents are interconnected and communicate (Klüver & Schmidt 1999).

Multi-agent simulations allow us to develop theory, demonstrate robust characteristics and observe the mechanisms behind unexpected, novel, emergent behavior (Brassel *et al.* 1997; Doreian & Stokman 1997; Freeman 1989; Troitzsch 1997; Wellman & Berkowitz 1988. We can investigate patterns that emerge from the interaction of explicitly defined states of individual agents and the causal processes that change these states over time (Deadman & Gimblett 1994; Fararo & Hummon 1994; Hanneman 1995; Itami *et al.* 2000) providing the capacity to study the complexity of these systems *in silico,* when real-world investigation is impractical, improbable, or impossible (Conte Hegselman & Terna 1997; Gilbert & Triotzsch 1999; Troitzsch 1998).

Clustering

Clustering appears as a phenomenon in diverse systems. It appears to be a common mechanism for coping with complexity. The formation of hierarchies of clusters reduces internal interactions and constrains behavior (Green 2002). Much has yet to be explained about how clusters emerge in multi-agent systems. Social groups can be described as clusters, alliances and networks, where common ideals, interests, and the like link individuals together (Lee 1980). They are formed from, and are maintained by, the patterns of connectivity and information exchange between members (Gilbert 1997). These patterns will influence the collective opinion of the network.

Previous studies by Stocker *et al*. (2001, 2002, 2003) focus on a binary choice of state, that is, either agreement (yes) or disagreement (no) about an issue. A significant research question concerns how a range of different opinions or ideas among a group of individuals connected by different network structures will affect collective or global opinion as members interact over time.

Individual states, social structure, and global opinion

In real-world situations, public opinion is usually diverse and spread across many different ideas, attitudes, and preferences. From time to time, there occurs a coalescing of public opinion towards main ideas that strongly resist changes over time (Schecter 2002), even though each individual makes a choice from several different ideas (Lomborg 1997). A relatively new area of research, Memetics, suggests that certain characteristics of ideas themselves influence selection (Aunger 2002; Blackmore 1999; Brodie 1996; Gladwell 1999; Lynch 1996; Marsden 2000).

Social network simulation research supports that group opinion is influenced by direct contact and communication between peers (Stocker *et al.* 2001, 2002, 2003). Ideas change depending on a susceptibility to attack from other ideas and the structure of connections between individuals (Hales 1998). Individual influence in the course of social transition is an important determinant of public opinion (Burt 1987). Public opinion change is dependent on the exchange of information between connected individuals (Nowak & Lewenstein 1996).

Social comparison stabilizes agreement of opinion (Granovetter 1978; Werner & Davis 1997) and depends on the nature of individuals and their relationships (Erickson 1988). However, diversity means that disagreement may also result in the formation of sub-groups whose members share similar points of view (Doreian & Stokman 1997). These influences have a critical effect on the dynamics of a social system (Nowak & Lewenstein 1996). Smaller disenfranchised or "fringe" groups can collect around radical opinions or ideas that are not representative of the majority.

In this study, A multi-agent simulation of specific network structures is used to represent the patterns of connection and communication between interacting agents. In what is essentially a network diffusion simulation (Valente 2005; see also Becker 1970; Rogers 1958, 1995) this simulation uses multiple instead of binary choices. The following questions are addressed: 1) how do individual states and membership of different network structures influence global opinion in a social network? and 2) do individual members tend to form a cluster around particular states or ideas so that sub-groups emerge?

Methods

In the simulation, 100 nodes are connected in three different network structures to represent patterns of connectivity and communication. Networks are static (that is, links and population remain the same) and network parameters are shown in Table 1. Each node is randomly initialized with a choice of state from 2 to 10 representing different issues or ideas. Each node is also randomly initialized with values for levels of influence and susceptibility (between 0.0 and 1.0). The result of interactions between nodes over time is observed to determine aggregation of nodes around particular states.

Table 1. Ranges of Values Assigned to Parameters in the Simulation for Each Network Type

	Hierarchy	Random	Scale-free
Network Parameters	2 to 10 (layers)	0.01 to 0.5 (connectivity)	0.0 to 1.0 (constant) 1.6 to 4.0 (exponent)

Nodes in the simulation change states asynchronously as do individuals in real world networks (Harvey & Bossomaier 1997; Cornforth *et al*. 2001, 2002). There are 10 runs of 1000 time steps for each combination of the network structure parameters.

The *expected number of nodes* (E) that adopt a given state at initialization will approximate to the *population* (N) divided by the *number of states* (S). For example, for 100 nodes with a choice of 5 states, we expect 20 nodes to adopt each state. Comparison of this expected value with the *average maximum number of nodes that adopt each state* after T iterations (time steps) provides a reliable measure of the relative degree to which nodes have adopted particular states as a result of their interaction. This comparison, expressed as an *Adoption Ratio* (AR) is defined by:

$$AR = \frac{\frac{1}{T}\left[\sum_{t=1}^{T}\max\left(n_1(t), n_2(t), \ldots n_i(t)\right)\right]}{\frac{N}{S}}$$

[Eq 1.]

Where $n_i(t)$ is the number of nodes adopting each state i at time t, N is the number of nodes in the population, T is the total number of time steps and S is the number of states available.

Results

Selected results below describe the behavior of hierarchy, random and scale-free networks and demonstrate that cohesion (the number of nodes in the same state) varies for the number of different states available to each node over time. As the number of states available increases, the maximum number of nodes adopting each state decreases, regardless of network structure.

There is a significant change of node "loyalty" to specific states where the maximum number of nodes adopting one or another state is not consistent. That is, aggregation of the nodes around specific states occurs to varying degrees. However, in each experiment only two or three states emerge as having the largest number of nodes adopting that state. This suggests that clustering is an emergent feature or principle of social structures independent of the parameters associated with network structure.

Of interest in hierarchy networks, is the critical change around a depth of 5 to 6 hierarchy layers (Figure 1). In the random network, there is evidence of critical behavior at a connectivity level of 0.25 to 0.3 (see Dunbar 1995; Wellman 1988) (Figure 1). Clustering of nodes around particular states is more evenly spread across the different states and node "loyalty" is less evident than in hierarchy networks.

As the scale-free constant (Z), the scale-free exponent (λ) and the number of states vary, the clustering of nodes around one to three states is less pronounced than for either the hierarchy or random structures. At a scale-free constant of 0.25, as the scale-free exponent and the number of states are varied, the maximum number of nodes adopting a state is only affected after the scale-free exponent reaches a value of 2.8. Interestingly, when the number of states is around 4, a significant reduction in the largest cluster size occurs for values of the scale-free exponent above 2.8. As the scale-free exponent moves above 2.8, a linear increase (approximate) in the maximum number of nodes adopting particular states occurs.

When the scale-free exponent is 2.2 and the scale-free constant and the number of available states are varied, the largest cluster is affected when the scale-free constant reaches 0.5. Again, when the number of states is around 4, a significant reduction in the largest cluster size occurs for values of the scale-free constant above 0.5 (Figure 1).

These preliminary results are somewhat expected. It is logical that when the number of choices available to a group is increased, the maximum number of individual’s choosing a particular state will necessarily decrease. Likewise, it is apparent that, at key values for each network’s structural parameters, there is change in the global behavior of the network - also an intuitive result.

Figure1. Maximum Cohesion in a Population of 100 Nodes for Hierarchy, Random, & Scale-free Networks

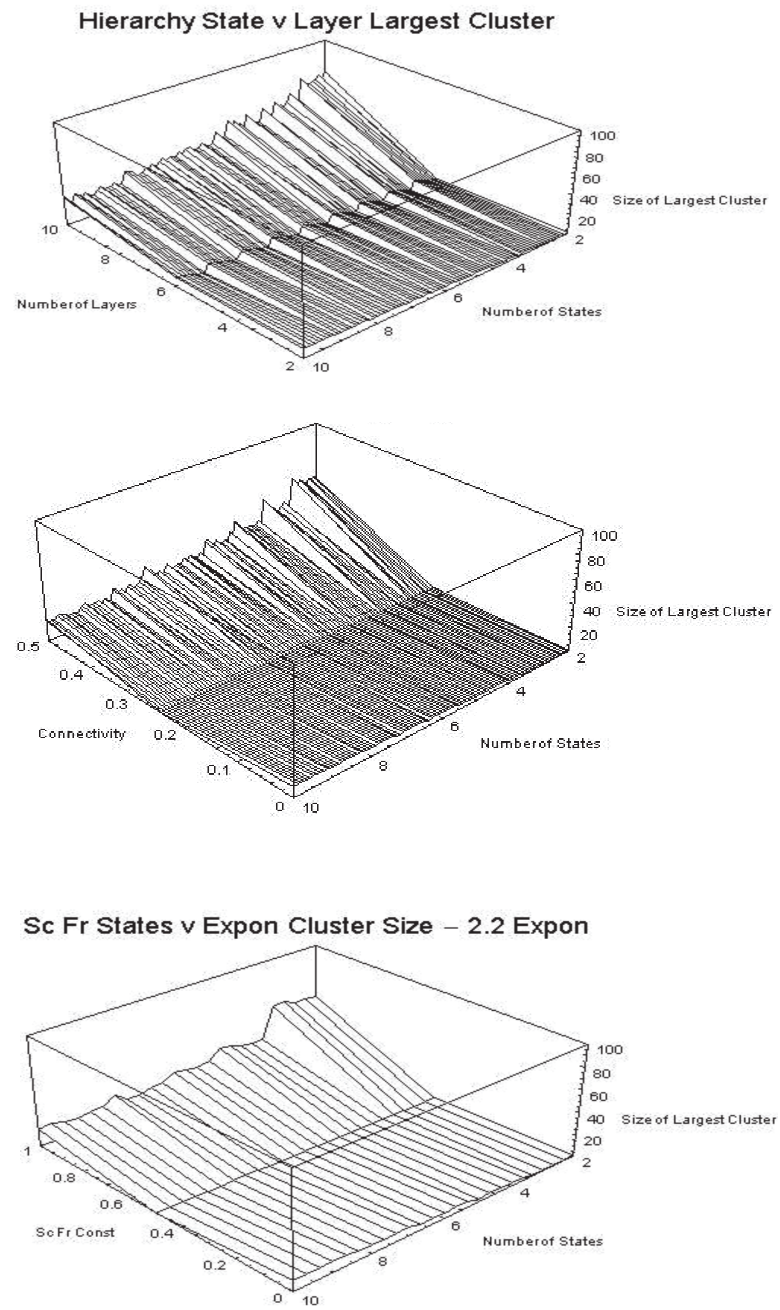

Figure 1. The surface plots show the relationship between the number of states that nodes can adopt and the key parameters of each network structure (viz, hierarchy layers, random connectivity and scale-free exponent/constant).

However when we examine the results from calculating the Adoption Ratio *AR* (Equation 1) over time, it is evident that counter-intuitive behavior is occurring. The behavior manifests as an increase in the *AR* as the number of states available increases. This occurs consistently for each of the hierarchy, random, and scale-free network structures.

In the hierarchy network, *AR* is consistently greater than 1.1 and varies between 1.1 and 2.2, showing clustering behavior away from the initialized state of the model. As the number of states is held constant and the depth of the hierarchy is varied, and the number of layers increases, *AR* remains fairly constant with a peak at 5 to 6 hierarchy layers. Counter-intuitively, as the number of layers is held constant and the number of states available is varied, the Adoption Ratio increases (Figure 2 – red squares).

For the random network, as the number of states is held constant and connectivity is varied *AR* remains within the range 1.1 to 1.8, with increased activity around critical connectivity of 0.25 to 0.30. When connectivity is held constant and the number of states is varied, *AR* increases with the number of states available with peaks at 5 and 7 states, a counter-intuitive result, indicating cluster size is dependent on number of states (Figure 2 – yellow triangles).

With the scale-free network structure, *AR* shows a steady increase as the number of available states increases with a peak at 6 states (Figure 2 – blue diamonds), indicating that cluster size increases with the number of available states. The similarity to hierarchy and random network behavior is evident.

Figure 2. Adoption Ratios by the Number of States in Hierarchy, Random, and Scale-free Network Structures

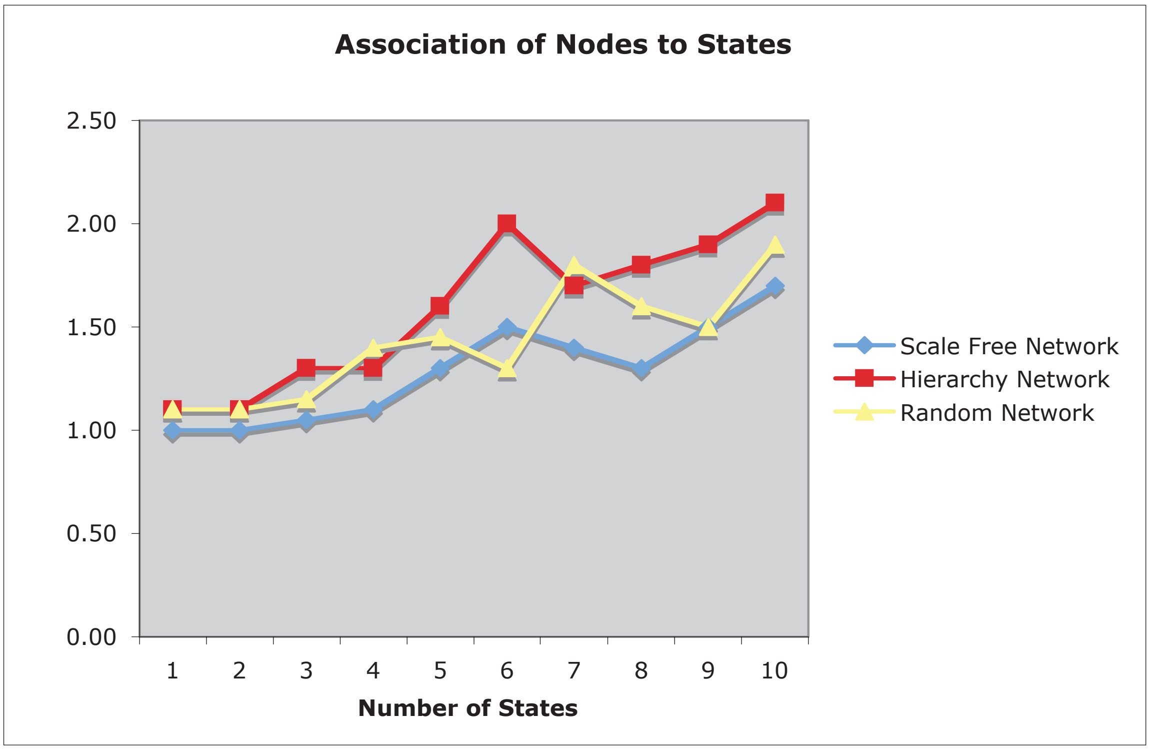

Figure 2. *Graph of the Adoption Ratio (AR) where the Y Axis shows AR and the X Axis shows the number of states in each of the hierarchy, random, and scale-free network structures. It shows the increase of AR with increasing number of states available.*

Discussion and Conclusions

Society comprises individuals who are connected by their involvement in work, social organisations, sporting clubs, religious communities, and so on. As members of more than one group they are influenced by the opinions, attitudes, and ideas of other members of the groups to which they belong. The boundaries of these groups often enclose a small to medium population of around 100 to 150 (Dunbar 1992, 1993; Wellman 1988). The manner in which members are connected will also vary.

This simulation suggests that network structure has an impact on the formation of public opinion in groups of social members that share common ideals, attitudes, or opinions. There is criticality with respect to parameters associated with network structure. In different social structures: (1) a majority of the population will change state from a large range of ideas to form aggregates, groups or clusters around on two to three preferred ideas, and, (2) clustering is dependent on the parameters associated with the patterns of connectivity between peer nodes (the structure of the networks). Clusters emerge as a result of a choice of states, although the maximum number of nodes that adopt each specific state reduces as the number of available states increases. This confirms an intuitive understanding that the more choices there are, the more difficult it is to make a choice. However, the *Adoption Ratio* (*AR)* demonstrated counter-intuitive behaviors. Regardless of the type of network, as the number of states available increases so did the Adoption Ratio, which demonstrates universality. One possible explanation (not yet confirmed) for this phenomenon is that some form of positive or reinforcing feedback is occurring amongst individuals in the social structure. As the number of nodes that adopt a specific state increases from the initial state above that expected, those nodes exert a converting force on the other nodes to which they are connected, regardless of network structure. However, there is a tension between this effect and the individual nodes' levels of influence and susceptibility. This tension provides the main constraint against the connected nodes changing state, thus explaining why the whole population is not converted. There is also criticality in respect of the parameters inherent in the different network structures. These factors suggest that there are general principles that apply to the formation of sub-groups within fixed populations, regardless of the structure to which they belong.

There are implications from this research for the management of change and survival of groups whose members will be connected to sources of different ideas, attitudes and opinions. Future research will focus on the behavior of dynamic networks where the network structure is influenced by the addition and deletion of nodes and links.

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